University of the West of England
BRISTOL

Bristol Business School

Academic Year: 10/11
Examination Period: January

Module Leader: John Paul Dunne
Module Code: Econometrics
Title of Module: UMEN3P-15-M

Examination Date: 12 January 2011
Examination Start time: 09:30
Duration of Examination: 2 Hours

Instructions to Candidates:

Candidates must answer THREE questions. The COMPULSORY question in Section A and any other two questions from Section B.

Candidates should use the answer books provided and write their student number and their name on each answer booklet used.

Materials supplied to the candidate will be:

Number of Examination Booklets per Examination	1
Number of sheets of Graph Paper size G3 (Normal)	0
Number of Pre-printed OMR (Multiple Choice Answer Sheet)	0

Additional Instructions to Invigilators:

University approved calculators may be used	Yes
Candidates permitted to keep Examination Question Paper	No
Material supplied by student allowed (must be collected with answer booklet) please specify:	No
Additional Specialised Material : Distribution Tables on Pages 10 – 13.	

Treasury tags & adhesive triangles will be supplied as standard

<u>Section A – Students MUST answer this question.</u>
This question is worth 40% of the overall mark.

Question One

Consider the following estimation results from Microfit:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is GC
 50 observations used for estimation from 1949 to 1998
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                        .0081152            .0044608            1.8192[.075]
 GY                         .59018             .077685            7.5971[.000]
 GP                        -.11219             .039605           -2.8327[.007]
 Z(-1)                    -.066153             .027551           -2.4011[.020]
*******************************************************************************
 R-Squared                     .66681   R-Bar-Squared                    .64508
 S.E. of Regression           .010850   F-stat.    F(  3,  46)   30.6867[.000]
 Mean of Dependent Variable   .023467   S.D. of Dependent Variable     .018213
 Residual Sum of Squares     .0054154   Equation Log-likelihood       157.3163
 Akaike Info. Criterion      153.3163   Schwarz Bayesian Criterion    149.4922
 DW-statistic                  1.1838
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   1)=   9.3009[.002]*F(   1,  45)=  10.2837[.002]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .72215[.395]*F(   1,  45)=   .65946[.421]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   2.8857[.236]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   1.2581[.262]*F(   1,  48)=   1.2389[.271]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

Where:
C: Constant
LC: Log of real consumers' expenditure in 1995 prices
LY: log of real personal disposable income in 1995 prices
LP: log of the consumer price index
GC=LC-LC(-1)
GY=LY-LY(-1)
GP=LP-LP(-1)
Z=LC-LY

Question One Cont'd…..

Required:

a) Briefly explain what the results tell us about the determination of consumption. **(25%)**

b) Briefly explain the meaning of the t ratios, the F-statistic, R-Squared, the DW statistic and the Residual Sum of Squares. **(25%)**

c) Briefly explain what diagnostic test A, diagnostic test B, and diagnostic test D are and what they tell us. **(20%)**

d) Consider the following estimates. **(30%)**

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is GC
 50 observations used for estimation from 1949 to 1998
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                          .30868            .27380              1.1274[.266]
 GY                         .61300            .079513             7.7093[.000]
 GP                        -.11198            .040374            -2.7734[.008]
 LC(-1)                    -.17379            .082656            -2.1025[.041]
 LY(-1)                     .14931            .068616             2.1760[.035]
 LP(-1)                   .0034151           .0062205             .54901[.586]
*******************************************************************************
 R-Squared                     .68074   R-Bar-Squared                   .64446
 S.E. of Regression           .010860   F-stat.    F(  5,  44)   18.7634[.000]
 Mean of Dependent Variable   .023467   S.D. of Dependent Variable     .018213
 Residual Sum of Squares     .0051891   Equation Log-likelihood       158.3835
 Akaike Info. Criterion      152.3835   Schwarz Bayesian Criterion    146.6474
 DW-statistic                  1.1532
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   1)=  12.6900[.000]*F(   1,  43)=  14.6252[.000]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .59297[.441]*F(   1,  43)=   .51607[.476]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   3.8397[.147]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   .73871[.390]*F(   1,  48)=   .71980[.400]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

Question One Cont'd………

And the following results:

```
               Wald test of restriction(s) imposed on parameters
*******************************************************************************
 Based on  OLS regression of GC on:
 C               GY               GP               LC(-1)           LY(-1)
 LP(-1)
 50 observations used for estimation from 1949 to 1998
*******************************************************************************
 Coefficients A1 to A6 are assigned to the above regressors respectively.
 List of restriction(s) for the Wald test:
                        a6=0; a4=-a5
*******************************************************************************
 Wald Statistic                 CHSQ( 2)=   1.9190[.383]
*******************************************************************************
```

Explain carefully what these results tell us (hint: consider how they relate to the results presented at the start of the question).

Section B - Answer 2 other questions from this section

Question Two

a) Write down an ARDL(1,1) model.

(10%)

b) Show how you can impose restrictions on your ARDL(1,1) model to derive at least 4 alternative static and dynamic nested models.

(40%)

c) Derive the static long run equilibrium of the equations in part a).

(20%)

d) Discuss the benefits of using an error correction model in applied econometric work

(30%)

Question Three

a) Define a stationary process. Explain what it means for variables to be I(0), I(1) and I(2).

(30%)

b) Explain in detail how to interpret the tables below and what they tell us about the variable LY (log of real personal disposable income in 1995 prices). GY is the first difference of LY.

(40%)

Unit root tests for variable LY

The Dickey-Fuller regressions include an intercept and a linear trend

46 observations used in the estimation of all ADF regressions.
Sample period from 1953 to 1998

	Test Statistic	LL	AIC	SBC	HQC
DF	-2.5852	115.3569	112.3569	109.6139	111.3293
ADF(1)	-3.3024	118.2592	114.2592	110.6019	112.8892
ADF(2)	-2.6337	118.5193	113.5193	108.9477	111.8068
ADF(3)	-2.5050	118.5404	112.5404	107.0544	110.4853
ADF(4)	-1.9249	119.4611	112.4611	106.0608	110.0635

95% critical value for the augmented Dickey-Fuller statistic = -3.5088
LL = Maximized log-likelihood AIC = Akaike Information Criterion
SBC = Schwarz Bayesian Criterion HQC = Hannan-Quinn Criterion

Unit root tests for variable GY

The Dickey-Fuller regressions include an intercept but not a trend

45 observations used in the estimation of all ADF regressions.
Sample period from 1954 to 1998

	Test Statistic	LL	AIC	SBC	HQC
DF	-5.2122	110.2021	108.2021	106.3954	107.5286
ADF(1)	-5.2508	111.7664	108.7664	106.0564	107.7562
ADF(2)	-4.4994	112.0494	108.0494	104.4361	106.7024
ADF(3)	-4.9082	114.0705	109.0705	104.5538	107.3867
ADF(4)	-3.4660	114.2007	108.2007	102.7807	106.1802

95% critical value for the augmented Dickey-Fuller statistic = -2.9271
LL = Maximized log-likelihood AIC = Akaike Information Criterion
SBC = Schwarz Bayesian Criterion HQC = Hannan-Quinn Criterion

c) Explain what cointegration is and how you would test for it using the Engle-Granger method. How would your answer change if you were dealing with more than 2 variables?

(30%)

Question Four

Consider the following model

$y_t = a + \beta x_t + u_t$

where $E(u_t) = 0$
$E(u_t) = \sigma^2$
$E(u_s, u_t) \neq 0 \ \forall s \neq t$

Required:

a) What problems would least squares estimators of this model have and what are the likely causes?

(40%)

b) Explain the methods by which you could test for first order serial correlation and then for higher order serial correlation.

(40%)

c) If you knew $u_t = 0.4\, u_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim IN(0, \sigma^2)$, then how would you proceed?

(20%)

Question Five

a) Explain why the errors in the regression equation need to have a common variance and what the implications are for OLS if they do not.

(40%)

b) Discuss how you might test for this problem in at least three different ways.

(40%)

c) In the linear regression model $Y_i = \alpha + \beta X_i + \varepsilon_i$ where ε_i are assumed to have a variance depending on Z_i, how would you choose between variance $(\varepsilon_i) = \sigma^2 Z_i$ and variance $(\varepsilon_i) = \sigma^2 Z_i^2$?

(20%)

Question Six

In the following linear model:

$$y_t = \alpha + \beta x_t + u_t$$

Required:

a) Derive the least squares estimator of β and show it is equivalent to the maximum likelihood estimator when $u_t \sim N(0, \sigma^2)$.

(40%)

b) Show that the least squares estimator of β is unbiased.

(30%)

c) Discuss how the results in sections a and b would be affected if a lagged dependent variable was introduced into the equation.

(30%)

Question Seven

a) An investigator estimated a model, with 4 coefficients including the constant, over the period 1950-80, which gave a residual sum of squares of 0.0012141, and then over the period 1950-85, which gave a residual sum of squares of 0.0014892.

   i. Undertake a predictive failure test to assess whether or not there is evidence of parameter instability in this model over the period 1950-85.

   (25%)

   ii. Show how dummy variables can be used to perform a test for structural stability. What do the coefficients and standard errors of the dummy variables represent?

   (25%)

b) Discuss the relationship between the Wald, LR and LM principles of constructing test statistics.

(50%)

Question Eight

Consider the model

$$Y_t = \alpha + \beta x_t + \delta z_t + \gamma w_t + \varepsilon_t \text{ where } i=1,\ldots N \text{ and } w_t = 2z_t$$

Required:

a) Explain in detail the problems of estimating the coefficients of this relationship using OLS.

(30%)

b) Discuss at least three ways in which you could detect multicollinearity and determine whether it was a problem.

(40%)

c) Discuss the pros and cons of dropping variables as a solution to multicollinearity.

(30%)

Question Nine

Consider the following model of supply and demand:

Demand: $Q_t^D = \beta_0 + \beta_1 P_t + \beta_2 W_t + u_{1t}$

Supply: $Q_t^S = \alpha_0 + \alpha_1 P_t + \alpha_2 Z_t + u_{2t}$

Where Q is quantity, P is price and W and Z are other relevant exogenous variables.

Required:

a) Derive the reduced form of the system stating your assumptions.

(50%)

b) Explain how you would estimate the structural parameters of the system and how your answer would be affected by an extra exogenous variable in the demand equation.

(50%)

END OF QUESTION PAPER

Table E-4 Cumulative student's t distribution*

$$F(t) = \int_{-\infty}^{t} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma(n/2)\sqrt{\pi n}\left(1 + \frac{x^2}{n}\right)^{(n+1)/2}} dx$$

n \ F	.75	.90	.95	.975	.99	.995	.9995
1	1.000	3.078	6.314	12.706	31.821	63.657	636.619
2	.816	1.886	2.920	4.303	6.965	9.925	31.598
3	.765	1.638	2.353	3.182	4.541	5.841	12.941
4	.741	1.533	2.132	2.776	3.747	4.604	8.610
5	.727	1.476	2.015	2.571	3.365	4.032	6.859
6	.718	1.440	1.943	2.447	3.143	3.707	5.959
7	.711	1.415	1.895	2.365	2.998	3.499	5.405
8	.706	1.397	1.860	2.306	2.896	3.355	5.041
9	.703	1.383	1.833	2.262	2.821	3.250	4.781
10	.700	1.372	1.812	2.228	2.764	3.169	4.587
11	.697	1.363	1.796	2.201	2.718	3.106	4.437
12	.695	1.356	1.782	2.179	2.681	3.055	4.318
13	.694	1.350	1.771	2.160	2.650	3.012	4.221
14	.692	1.345	1.761	2.145	2.624	2.977	4.140
15	.691	1.341	1.753	2.131	2.602	2.947	4.073
16	.690	1.337	1.746	2.120	2.583	2.921	4.015
17	.689	1.333	1.740	2.110	2.567	2.898	3.965
18	.688	1.330	1.734	2.101	2.552	.2878	3.922
19	.688	1.328	1.729	2.093	2.539	2.861	3.883
20	.687	1.325	1.725	2.086	2.528	2.845	3.850
21	.686	1.323	1.721	2.080	2.518	2.831	3.819
22	.686	1.321	1.717	2.074	2.508	2.819	3.792
23	.685	1.319	1.714	2.069	2.500	2.807	3.767
24	.685	1.318	1.711	2.064	2.492	2.797	3.745
25	.684	1.316	1.708	2.060	2.485	2.787	3.725
26	.684	1.315	1.706	2.056	2.479	2.779	3.707
27	.684	1.314	1.703	2.052	2.473	2.771	3.690
28	.683	1.313	1.701	2.048	2.467	2.763	3.674
29	.683	1.311	1.699	2.045	2.462	2.756	3.659
30	.683	1.310	1.697	2.042	2.457	2.750	3.646
40	.681	1.303	1.684	2.021	2.423	2.704	3.551
60	.679	1.296	1.671	2.000	2.390	2.660	3.460
120	.677	1.289	1.658	1.980	2.358	2.617	3.373
∞	.674	1.282	1.645	1.960	2.326	2.576	3.291

* This table is abridged from the "Statistical Tables" of R. A. Fisher and Frank Yates published by Oliver & Boyd, Ltd., Edinburgh and London, 1938. It is here published with the kind permission of the authors and their publishers.

Table E-5[1] Durbin-Watson statistic (d). Significance points of d_L and d_U: 5%

n	k' = 1		k' = 2		k' = 3		k' = 4		k' = 5	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93	0.62	2.15
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90	0.67	2.10
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87	0.71	2.06
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85	0.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81	0.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80	0.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79	0.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	0.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	0.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78

n = number of observations.
k' = number of explanatory variables.

[1] This Table is reproduced from *Biometrika*, vol. 41, p. 173, 1951, with the permission of the Trustees.

Table E-3 Cumulative chi-square distribution*

$$F(u) = \int_0^u \frac{x^{(n-2)/2} e^{-x/2}\, dx}{2^{n/2}\Gamma(n/2)}$$

n \ F	.005	.010	.025	.050	.100	.250	.500	.750	.900	.950	.975	.990	.995
1	$.0^4393$	$.0^3157$	$.0^3982$	$.0^2393$	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	.0717	.115	.216	.352	.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8
4	.207	.297	.484	.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9
5	.412	.554	.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7
6	.676	.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5
7	.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7

* This table is abridged from "Tables of percentage points of the incomplete beta function and of the chi-square distribution," *Biometrika*, Vol. 32 (1941). It is here published with the kind permission of its author, Catherine M. Thompson, and the editor of *Biometrika*.

Table 14.1 Critical Values for Unit Root Tests

Sample Size	K-Test 1%	K-Test 5%	t-Test 1%	t-Test 5%	F-Test[a] 1%	F-Test[a] 5%
AR (1)						
25	−11.9	−7.3	−2.66	−1.95		
50	−12.9	−7.7	−2.62	−1.95		
100	−13.3	−7.9	−2.60	−1.95		
250	−13.6	−8.0	−2.58	−1.95		
500	−13.7	−8.0	−2.58	−1.95		
∞	−13.8	−8.1	−2.58	−1.95		
AR (1) with constant						
25	−17.2	−12.5	−3.75	−3.00		
50	−18.9	−13.3	−3.58	−2.93		
100	−19.8	−13.7	−3.51	−2.89		
250	−20.3	−14.0	−3.46	−2.88		
500	−20.5	−14.0	−3.44	−2.87		
∞	−20.7	−14.1	−3.43	−2.86		
AR (1) with constant and trend						
25	−22.5	−17.9	−4.38	−3.60	7.24	10.61
50	−25.7	−19.8	−4.15	−3.50	6.73	9.31
100	−27.4	−20.7	−4.04	−3.45	6.49	8.73
250	−28.4	−21.3	−3.99	−3.43	6.34	8.43
500	−28.9	−21.5	−3.98	−3.42	6.30	8.34
∞	−29.5	−21.8	−3.96	−3.41	6.25	8.27

[a] $K = T(\hat{\rho} - 1)$, $t = (\hat{\rho} - 1)/SE(\hat{\rho})$ and F-test is for $\gamma = 0$ and $\rho = 1$ in $y_t = \alpha + \gamma t + \rho y_{t-1} + u_t$.
Source: W. A. Fuller, *Introduction to Statistical Time Series* (New York: Wiley, 1976), p. 371 for the K-test and p. 373 for the t-test; D. A. Dickey and W. A. Fuller, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica*, Vol. 49, No. 4, 1981, p. 1063 for the F-test.

Table 14.2 Critical Values (5%) for the Cointegration Tests

n	T	CRDW	DF	ADF[a]
2	50	0.78	−3.67	−3.29
	100	0.39	−3.37	−3.17
	200	0.20	−3.37	−3.25
3	50	0.99	−4.11	−3.75
	100	0.55	−3.93	−3.62
	200	0.39	−3.78	−3.78
4	50	1.10	−4.35	−3.98
	100	0.65	−4.22	−4.02
	200	0.48	−4.18	−4.13
5	50	1.28	−4.76	−4.15
	100	0.76	−4.58	−4.36
	200	0.57	−4.48	−4.43

[a] $CRDW = \sum (\hat{u}_t - \hat{u}_{t-1})^2 / \sum \hat{u}_t^2$, CRDW means "cointegrating regression Durbin–Watson" statistic; DF = t-test for $\alpha = 0$ in $\Delta \hat{u}_t = \alpha \hat{u}_{t-1} + \eta_t$; ADF = t-test for $\alpha = 0$ in $\Delta \hat{u}_t = \alpha \hat{u}_{t-1} + \sum_{1}^{p} \phi_i \Delta \hat{u}_{t-1} + \eta_t$. In all these tests $\hat{u}_t$ is the residual from the cointegrating regression.
Source: R. F. Engle and S. Yoo, "Forecasting and Testing in Cointegrated Systems," *Journal of Econometrics*, Vol. 35, 1987.

Table E-7 F distribution, upper 5% points ($F_{0.95}$).[1]

Degrees of freedom for numerator (columns); Degrees of freedom for denominator (rows)

	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Interpolation should be performed using reciprocals of the degrees of freedom.

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